Non-supersymmetric deformation of the Klebanov-Strassler model and the related plane wave theory

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We find a regular analytic 1st order deformation of the Klebanov-Strassler background. From the dual gauge theory point of view the deformation describes supersymmetry soft breaking gaugino mass terms. We calculate the difference in vacuum energies between the supersymmetric and the non-supersymmetric solutions and find that it matches the field theory prediction. We also discuss the breaking of the $U(1)_R$ symmetry and the space-time dependence of the gaugino bilinears two point function. Finally, we determine the Penrose limit of the non-supersymmetric background and write down the corresponding plane wave string theory. This string describes "annulons"-heavy hadrons with mass proportional to large global charge. Surprisingly the string spectrum has two fermionic zero modes. This implies that the sector in the non-supersymmetric gauge theory which is the dual of the annulons is supersymmetric.

1 Introduction

Since the formulation of the AdS/CFT conjecture [1], [2], [3] (see [4] for a review) there has been great progress in the study of theories with less supersymmetries and not necessarily conformal. There are several approaches one can use to break the $\mathcal{N} = 4$ supersymmetry down to $\mathcal{N} = 2$ or $\mathcal{N} = 1$.

A few years ago two important examples of supergravity duals of $\mathcal{N} = 1$ gauge theories have been provided by [5] and [6] (see [7] and [8] for recent reviews). The Maldacena-Nunez (MN) background consists of NS5-branes wrapped on an S^2 and based on the solution of [9]. The supergravity dual of Klebanov-Strassler (KS) involves D5 branes wrapped around a shrinking S^2. The metric has a standard D3-form with the 6d deformed conifold being the transversal part of the 10d space.

Non-supersymmetric deformations of the MN background have been studied by number of authors. In [10] the supersymmetry was broken completely by giving masses for some of the scalar fields. It was argued that the deformed non-supersymmetric background is guaranteed to be stable, since the original dual gauge theory had a mass gap. On other hand, the authors of [11] used the solution of [12] to study the supersymmetry breaking by the inclusion of a gaugino mass term and a condensate. Evidently, the global symmetry remains unbroken under this deformation.

Our main goal is to find a non-singular, non-supersymmetric deformation of the KS solution, which preserves the global symmetries of the original background and to study the Penrose limit of the new solution. The problem has been already attacked by different authors [13],[14]. The authors of [14] suggested a computational technique for studying the non-supersymmetric solution. The technique is based on the modification of the first order BPS equations, so that we might continue to use a superpotential even for a non-supersymmetric solution. In short, one obtains a set of sixteen 1st equations and one zero-order constraint instead of eight standard 2nd order differential equations.

In this paper (see [15] for more comprehensive discussion) we determine and describe a regular *analytic* solution of the 1st order equations similar to those appearing in [14]. We note that these equations are significantly simplified once we properly redefine the radial coordinate. (The equations transform non-trivially under the coordinate redefinition since one has to apply the "zero-energy" constraint, which removes the "gauge freedom" of the coordinate transformation). We also demonstrate how part of the 1st order equations can be re-derived using the usual 2nd order IIB equations of motion.

Our solution preserves the global symmetry and therefore describes a deformation corresponding to the inclusion of mass terms of the

two gaugino bilinears in the dual gauge theory.

We construct a Penrose limit (see [16], [17], [18], [19], [20] and [21]) of our non-supersymmetric KS background and obtain a pp-wave metric and a complex 3-form which are very similar to the PL limit [22] of the supersymmetric solution.

We also quantize the light-cone string Hamiltonian and determine the masses of the bosonic and fermionic modes. These masses, though different from the supersymmetric case, still obey the relation that the sum of the mass squared is the same for bosonic and fermionic modes. Again the string describes kinematics and excitations of heavy hadrons (called "annulons" [22]) with masses proportional to a large global symmetry charge $M_{annulon} = m_0 J$. The only difference between them and those of [22] is a modification of m_0. A surprising feature of the string spectrum is that, like in the Penrose limit of the KS background, here as well, there are two fermionic zero modes. In the dual field theory this implies that even though the full theory is non-supersymmetric, the sector of states with large J charge admits supersymmetry. It is conceivable that in this limit of states of large mass the impact of the small gaugino mass deformations is washed away.

The authors of [23] used the solution of [13] to take the PL. The IR expansion of the fields given in [13] differs, however, from our solution (see later) and therefore the pp-wave background of [23] is also slightly different from the metric we have derived.

2 The Klebanov-Strassler model and beyond

Before reviewing the main features of the KS solution it will be worth to write down the type IIB equation of motion for a case of a constant dilaton ($e^\Phi = g_s$), a vanishing axion ($C_0 = 0$) and with the 10d metric and the 5-form flux having the structure of the D3-brane solution, namely:

$$ds^2 = h^{-1/2}\mathbf{d}x_\mu^2 + h^{1/2}\mathbf{s}_{M_6}^2 \qquad (1)$$

and

$$\tilde{F}_5 = \frac{1}{g_s}(1 + \star_{10})\mathbf{d}h^{-1} \wedge \mathbf{d}x_0 \wedge \ldots \wedge \mathbf{d}x_3, \quad (2)$$

where M_6 is a 6d Ricci flat transversal space and the harmonic function h depends only on the coordinates on M_6. We will denote the Hodge dual on M_6 by $\star_6$. In order to find the connection between the 3-forms and the warp function h we have to use the 5-form equation. We end up with:

$$\tilde{F}_5 = B_2 \wedge F_3 + \star_{10}(B_2 \wedge F_3)$$
$$\text{or} \quad \mathbf{d}h = -g_s \star_6 (B_2 \wedge F_3), \qquad (3)$$

where the 2nd equation is the integrated version of the first. Next we consider the 3-forms equations. Applying (3) and the relation between F_5 and $\tilde{F}_5$ we get:

$$\mathbf{d}\left[h^{-1}\left(\star_6 F_3 + \frac{1}{g_s}H_3\right)\right] = 0 \qquad (4)$$

and similarly for H_3. In deriving this result we have used the fact that all the forms have their legs along the 6d space. Finally, calculating the Ricci scalar of the metric (1) we re-write the metric equation of motion:

$$\mathbf{d}\star_6 \mathbf{d}h = \frac{1}{2}\left[H_3 \wedge \star_6 H_3 + g_s^2 F_3 \wedge \star_6 F_3\right]. \quad (5)$$

The equations we have written (3,4,5) as well as the the dilaton and the axion equations are easily solved by requiring that:

$$\star_6 F_3 = -g_s^{-1}H_3 \quad \text{and} \quad \star_6 H_3 = g_s F_3. \quad (6)$$

In this case the complex form $G_3 \equiv F_3 + \frac{i}{g_s}H_3$ is imaginary self dual $\star_6 G_3 = iG_3$.

Note that the equation for h is a first order differential equation, even though the solution is not supersymmetric in general.

The most important example of the supersymmetric solution is the Klebanov-Strassler model [6], where the 6d manifold is the deformed conifold space. The $\mathbf{M}$ fractional D5-branes wrapping the shrinking S^2 are introduced through the RR 3-form and on using the duality relations (6) one may also find the NS 3-form:

$$H_3 = g_s\mathbf{M}\mathbf{d}\left[f(\tau)g^1 \wedge g^2 + k(\tau)g^3 \wedge g^4\right],$$
$$F_3 = \mathbf{M}\Big[g^5 \wedge g^3 \wedge g^4 +$$
$$+\mathbf{d}\left(F(\tau)g^1 \wedge g^3 + g^2 \wedge g^4\right)\Big] \qquad (7)$$

where τ is the radial coordinate and the functions $f(\tau), k(\tau)$ and $F(\tau)$ satisfy a set of three

first order differential equations [6]. This set has three dimensional space of solutions. Using the complex structure of the deformed conifold space [24] the complex form $G_3 = F_3 + \frac{i}{g_s}H_3$ can be identified for the Klebanov-Strassler solution as a regular $(2,1)$ form. There are also two additional solutions corresponding to a $(0,3)$ form which breaks the supersymmetry and diverges at $\tau \to \infty$ and a $(2,1)$ form which is singular at $\tau = 0$.

The dual field theory realized on the world-volume of the **N** physical and **M** fractional D3-branes is a 4d $\mathcal{N} = 1$ supersymmetric $SU(\mathbf{N} + \mathbf{M}) \times SU(\mathbf{N})$ gauge theory with a $SU(2) \times SU(2)$ global symmetry inherited from the conifold isometries. The gauge theory is coupled to two bi-fundamental chiral multiplets A and B, which transform as a doublet of one of the $SU(2)$'s each and are inert under the second $SU(2)$. This theory is believed to exhibit a cascade of Seiberg dualities reducing in the deep IR to pure $SU(\mathbf{M})$. On the supergravity side **M** is fixed by the charge of the RR 3-form, while **N** is encoded in the UV behavior of the 5-form. The sum of the gauging couplings is constant and the logarithmic running of the difference is determined by the NS 2-form.

Similarly to pure $SU(\mathbf{M})$ the theory confines. This is evident by virtue of the fact that the warp factor approaches a constant value $h_0 \sim a_0$ at $\tau \to 0$ and therefore the tension of the confining strings does not diverge. This conclusion is valid only for a non-zero value of the deformation parameter ϵ, since $a_0 \sim \epsilon^{-8/3}$. Note also that for $\epsilon \neq 0$ the $U(1)_R$ conifold symmetry is broken down to $\mathbf{Z}_2$. This is the symmetry preserved by the gaugino bilinear $\mathrm{Tr}\lambda\lambda(x)$. In the supergravity dual this gauge theory operator is associated with the form $C_2 = C_2^{RR} + iB_2^{NS}$ [25]. Subtracting the asymptotic value of $G_3 = \mathbf{d}C_2$ we find at $\tau \to \infty$:

$$\Delta G_3 \approx \frac{\mathbf{M}}{2}\tau e^{-\tau}\omega_3, \quad \Delta C_2 \approx -\frac{\mathbf{M}}{2}\tau e^{-\tau}\omega_2,$$
$$\omega_3 = \mathbf{d}\omega_2, \quad \omega_2 = \Big[(g^1 \wedge g^3 + g^2 \wedge g^4) +$$
$$+ig_s \left(g^1 \wedge g^2 - g^3 \wedge g^4\right) \Big], \quad (8)$$

where we write only the polarization along $T_{1,1}$ and we see that ΔC_2 transforms under $U(1)_R$ by the same phase as $\mathrm{Tr}(\lambda\lambda)$. Moreover, ΔG_3 has an asymptotic behavior we would expect from a scalar operator of dimension 3 and a non-zero VEV, namely: $\Delta G_3 =$

$\frac{1}{2}\mathbf{M}\frac{m^3}{r^3}\ln\frac{r^3}{m^3}\omega_3$, where the deformation parameter is related to the 4d mass scale through $m \sim \epsilon^{2/3}$.

Finally, we will recall the identification of supergravity fields with gauge theory operators. In order to find this correspondence one writes the most general $SU(2) \times SU(2)$ invariant background ansatz, which includes the supersymmetric KS solution:

$$ds^2 = 2^{1/2}3^{3/4}\Big[e^{-5q+2Y}(dx_\mu)^2 +$$
$$+\frac{e^{3q-8p}}{9}(d\tau^2 + g_5^2) + \frac{e^{3q+2p+y}}{6}(g_1^2 + g_2^2) +$$
$$+\frac{e^{3q+2p-y}}{6}(g_3^2 + g_4^2)\Big], \quad \Phi = \Phi(\tau), \quad (9)$$

with the 3-forms are given by (7) and the 5-form by (2). This general ansatz includes both the conformal solution with a singular geometry ($y = \tilde{f} - \tilde{k} = 0$) and the non-conformal case with regular deformed conifold ($y, \tilde{f} - \tilde{k} \neq 0$). Here $\tilde{f}, \tilde{k}$ and $\tilde{F}$ are the rescaled KS functions: $\tilde{f} = -2Pg_s f$, $\tilde{k} = -2Pg_s k$, $\tilde{F} = 2PF$ and the constant P is related to the number of fractional branes: $P = \frac{1}{4}\mathbf{M}l_s^2$. Note that for the given structure of the 3-form F_3 the integral $\int_{S_3} F_3$ does not depend on $\tilde{F}(\tau)$. Moreover, the NS-NS 3-form has the same structure as in the KS solution as dictated by the equation for a vanishing axion $H_3 \wedge F_3 = 0$.

Integration of the type IIB Lagrangian over the angular and the world-volume coordinates yields a 1d effective action:

$$S \sim \int d\tau \left(-\frac{1}{2}G_{ij}\dot{\phi}^i\dot{\phi}^j - V(\phi)\right), \quad (10)$$

and we refer the reader to [26],[27],[24], [28] for an explicit form of the metric and the potential $V(\phi)$. There is also a "zero-energy" constraint $\frac{1}{2}G_{ij}\dot{\phi}^i\dot{\phi}^j - V(\phi) = 0$. This Lagrangian admits a superpotential

$$V = \frac{1}{8}G^{ij}\partial_i W \partial_j W \quad \text{for}$$
$$W = -3e^{4Y+4p-4q}\cosh y -$$
$$-2e^{4Y-6p-4q} - 3\sqrt{3}e^{4Y-10q}\tilde{L} \quad (11)$$

and for supersymmetric solutions the second order equations of motion can be reduced to the first order ones:

$$\dot{\phi}^i = \frac{1}{2}G^{ij}\partial_j W. \quad (12)$$

The potential appearing in the action has an $\mathcal{N} = 1$ critical point corresponding to the

conformal background $AdS_5 \times T_{1,1}$ generated by physical D3-branes in absence of fractional branes ($P = 0$). Expanding the potential around the critical point and using the mass/dimension formula $\Delta = 2 + \sqrt{4 + m^2}$ one obtains the dimensions of the fields, which now can be identified with various gauge theory operators [29], [28]. Here we list two of them (both with $\Delta = 3$):

$$\xi_2 \sim -F + \tfrac{k-f}{2} \to \mathrm{Tr}\left(W_{(1)}^2 + W_{(2)}^2\right),$$
$$y \to \mathrm{Tr}\left(W_{(1)}^2 - W_{(2)}^2\right)$$

There are also two massless fields. $s = f + k$ is associated with a marginal direction in the CFT and the corresponding operator is $\mathrm{Tr}\left(F_{(1)}^2 - F_{(2)}^2\right)$. Similarly, the dilaton Φ corresponds to $\mathrm{Tr}\left(F_{(1)}^2 + F_{(2)}^2\right)$.

In this paper we will focus on the non-supersymmetric deformation of the KS background by introducing mass terms of the gaugino bilinears associated with both ξ_2 and y. The former field is related to the SUGRA 3-forms and the latter is responsible for a deformation of the 6d metric. The expected UV behavior of the fields in the background deformed by the masses is $g(\tau)e^{-\tau/3}$, where $g(\tau)$ is a polynomial in τ.

3 Non-supersymmetric extension of KS

We start this section with a brief review of the method proposed by [14] (see also [30], [31], [32], [33], [34] and [35]) to study first order non-supersymmetric deformations of the KS background still making use of the superpotential. We expand the fields around a given supersymmetric solution derived from the superpotential $\phi^i = \phi_0^i + \delta \cdot \bar{\phi}^i + O(\delta^2)$. Define new functions:

$$\xi_i = G_{ij}(\phi_0)\left(\tfrac{d\bar{\phi}^j}{d\tau} - M_k^j(\phi_0)\bar{\phi}^k\right) \quad \text{where}$$
$$M_k^j = \tfrac{1}{2}\tfrac{\partial}{\partial\phi^k}\left(G^{jl}\tfrac{\partial W}{d\phi^l}\right). \tag{13}$$

Now one might represent the linearized equations of motion as a "double" set of first order equations (we refer the reader to [14] for the proof):

$$\frac{d\xi_i}{d\tau} + \xi_j M_i^j = 0, \quad \frac{d\bar{\phi}^i}{d\tau} - M_j^i\bar{\phi}^j = G^{ik}\xi_k \tag{14}$$

while the zero-energy condition can be rephrased as $\xi_k \partial_\tau \bar{\phi}^k = 0$.

An important remark is in order. One can use various definitions for the radial coordinate in the 1d effective action. This ambiguity is removed by applying the zero-energy constraint. The explicit form of the 1st order equations (14) is highly dependent on the radial coordinate choice. In our paper we will fix this "gauge freedom" by requiring that even in the deformed solution the $G_{\tau\tau}$ and G_{55} entries of the metric will remain equal exactly as in the supersymmetric case. We will see that with this choice the set of the equations (14) possesses an analytic solution. On the contrary the radial coordinate ($\tau_\star$) of [14] is related to our coordinate (τ) via $d\tau_\star = e^{4\bar{p}-4\bar{q}}d\tau$. Note, however, that since both $\bar{p}(\tau)$ and $\bar{q}(\tau)$ are expected to vanish at $\tau \to 0$ and $\tau \to \infty$, the deep UV and IR expansions of the fields have to be the same in terms of τ and $\tau_\star$.

Let us first consider the equations of motion for ξ_i's [1]. Throughout this paper we will be interested in a solution satisfying: $\xi_Y = \xi_p = \xi_q = 0$. Under this assumption we get:

$$\dot{\xi}_y = \xi_y \cosh y_0 + 2e^{2y_0}(2P - \tilde{F}_0)\xi_{\tilde{f}} -$$
$$-2e^{-2y_0}\tilde{F}_0\xi_{\tilde{k}}, \qquad \dot{\xi}_{\tilde{f}+\tilde{k}} = 0,$$
$$\dot{\xi}_{\tilde{F}} = -\cosh(2y_0)\xi_{\tilde{f}-\tilde{k}} - \sinh(2y_0)\xi_{\tilde{f}+\tilde{k}},$$
$$\dot{\xi}_\Phi = \left(e^{2y_0}(2P - \tilde{F}_0)\xi_{\tilde{f}} + e^{-2y_0}\tilde{F}_0\xi_{\tilde{k}}\right) -$$
$$-\tfrac{\tilde{k}_0-\tilde{f}_0}{2}\xi_{\tilde{F}}, \qquad \dot{\xi}_{\tilde{f}-\tilde{k}} = -\xi_{\tilde{F}}, \tag{15}$$

where $\xi_{\tilde{f}\pm\tilde{k}} = \xi_{\tilde{f}} \pm \xi_{\tilde{k}}$. We have $\xi_{\tilde{f}+\tilde{k}} = X$ for constant X and from the equations for $\xi_{\tilde{f}-\tilde{k}}$ and $\xi_{\tilde{F}}$ we obtain a 2nd order differential order equation for $\xi_{\tilde{f}-\tilde{k}}$. This equation has a two dimensional space of solutions. However, solving for ξ_y, plugging the result into the zero-energy constraint $\xi_i\dot{\phi}_0^i = 0$ and requiring also regularity at $\tau \to 0$ we pick up a unique simple solution $\xi_{\tilde{f}-\tilde{k}}(\tau) = X\cosh\tau$ and therefore:

$$\xi_{\tilde{F}} = -X\sinh\tau, \quad \dot{\xi}_\Phi = 0,$$
$$\xi_y = 2PX(\tau\cosh\tau - \sinh\tau), \tag{16}$$

Having determined the explicit form of ξ_i's we can consider the equations for the fields $\bar{\phi}^i$'s. For $\bar{y}$ we get:

$$\dot{\bar{y}} + \cosh(y_0)\bar{y} = \frac{2}{3}e^{4q_0-4p_0-4Y_0}\xi_y. \tag{17}$$

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[1] We will set $g_s = 1$ throughout this section

Using the result for ξ_y and substituting the expressions for $q_0(\tau)$, $p_0(\tau)$ and $Y_0(\tau)$ we may solve for $\bar{y}(\tau)$ fixing an integration constant by requiring regularity at $\tau \to 0$ (see [15]). In this review we will need an asymptotic behavior of $\bar{y}(\tau)$ at $\tau \to \infty$:

$$\bar{y} \approx \mu\left(\tau - \frac{5}{2}\right)e^{-\tau/3} + Ve^{-\tau} + \ldots, \quad (18)$$

where μ is a deformation parameter proportional to X and V is a numerical constant proportional to μ. Note that μ is a dimensionless parameter. Using the result for $\bar{y}(\tau)$ and the fact that $\xi_p = 0$ we may find the solution for $\bar{p}(\tau)$. We refer the reader to the original paper [15] for a full analytic solution for $p(\tau)$ and other fields (in particular it appears that $\bar{\Phi} = 0$). Here we will only review the derivation of the results for the 3-form fields. Using the expressions for $\xi_{\tilde{f} \pm \tilde{k}}$ and $\xi_{\tilde{F}}$, passing from $\tilde{f}$, $\tilde{k}$ and $\tilde{F}$ to f, k and F we obtain and recalling that $\bar{\Phi} = 0$:

$$\dot{\bar{f}} + e^{2y_0}\bar{F} - 2\dot{f}_0\bar{y} = -\frac{2X}{2P}h_0(\cosh\tau - 1)$$
$$\dot{\bar{k}} - e^{-2y_0}\bar{F} + 2\dot{k}_0\bar{y} = \frac{2X}{2P}h_0(\cosh\tau + 1)$$
$$\dot{\bar{F}} - \frac{1}{2}(\bar{k} - \bar{f}) = -\frac{2X}{2P}h_0\sinh\tau. \quad (19)$$

Before discussing the explicit solution of this system it is worth to re-derive these equations using the 2nd order type IIB equations of motion. In the most general ansatz preserving the global symmetry the 5-form $\tilde{F}_5$ is given by

$$\tilde{F}_5 = \frac{1}{g_s}(1 + \star_{10})\mathbf{d}\varphi \wedge \mathbf{d}x_0 \wedge \ldots \wedge \mathbf{d}x_3, \quad (20)$$

where $\varphi = \varphi(\tau)$. Supersymmetry requires $\varphi = h^{-1}$ (see [36] and [37]) , but it does not necessarily hold in a non-supersymmetric case. In what follows we will demonstrate how assuming that $\dot{\Phi} = 0$ and $\varphi = h^{-1}$ one may reproduce (19) from the usual 2nd order 3-forms equations of motion. Indeed, under these assumptions the type IIB 3-forms equations reduce to (4). Let us expand (4) around the supersymmetric KS solution. Note that the expansion includes also $\star_6$ due to the deformation of the 6d space. We will denote the modified Hodge star operation by $\star_6 = \star_6^{(0)} + \bar{\star}_6$, where $\star_6^{(0)}$ corresponds to the supersymmetric configuration. After some algebra the linearized RR

3-form equation reduces to:

$$\mathbf{d}Z_3 = 0, \quad \text{where}$$
$$Z_3 = \frac{1}{g_s}\bar{H}_3 + \star_6^{(0)}\bar{F}_3 + \bar{\star}_6 F_3^{(0)}. \quad (21)$$

where $F_3^{(0)}$ is the RR 3-form in the KS background. Similarly, from the NSNS 3-form equation we have:

$$\mathbf{d}\star_6 Z_3 = 0. \quad (22)$$

Comparing this with (19) we see that the r.h.s. of (19) is exactly the components of the closed (and co-closed) form Z_3. Notice that having $Z_3 \neq 0$ necessary means that the complex form $G_3 = F_3 + \frac{i}{g_s}H_3$ is not imaginary self dual and therefore the supersymmetry is broken [36], [37]. The most general solution of (21) and (22) has 3 integration constants and it appears in [15]. In particular, it turns out that the 3-form on the r.h.s. of (19) corresponds to the divergent $(0, 3)$-form we have mentioned in the discussion following (7). Remarkably, this is the only solution for Z_3, which is consistent with $\dot{\Phi} = 0$. To find the solution for $\bar{F}(\tau)$, $\bar{f}(\tau)$ and $\bar{k}(\tau)$ note that the homogeneous part of (19) reduces to an equation of the form $dZ_3 = d\star_6 Z_3 = 0$ and as we have already mentioned the related 3-parameter solution appears in [15]. Using this solution we may easily find the solution of the three inhomogeneous equations (see [15]). In the UV we have:

$$\bar{F}(\tau) \approx \mu\left(\frac{3}{4}\tau - 3\right)e^{-\tau/3} + \left(\frac{3}{2}V + V'\right)e^{-\tau}$$
$$\bar{f}(\tau) \approx -\frac{27}{16}\mu e^{-\tau/3} + \left(\frac{V}{2} + V'\right)e^{-\tau} + \ldots$$
$$\bar{k}(\tau) \approx \frac{27}{16}\mu e^{-\tau/3} - \left(\frac{V}{2} + V'\right)e^{-\tau} + \ldots,$$

where V' is a constant proportional to μ.

Let us summarize. The deformation is controlled by the single parameter μ and all the fields have a regular behavior in the UV and in the IR. There are two non-normalizable modes. The first one is $y(\tau)$ and it is related to the deformation of the 6d metric. The second one is ξ_2 and it is associated with the 3-forms. In the UV we have:

$$\xi_2 \sim -F + \frac{k - f}{2} \approx -\frac{3}{4}\mu\left(\tau - \frac{25}{4}\right)e^{-\tau/3}. \quad (23)$$

Both $y(\tau)$ and ξ_2 have dimension $\Delta = 3$ which matches perfectly with the asymptotic behavior of the fields. In the dual gauge theory

these operators are dual to the gaugino bilinears. The deformation also involves other fields like $s = f + k$ with a normalizable behavior at $\tau \to \infty$. For example, $s \approx e^{-4\tau/3}$ as expected for an operator with $\Delta = 4$.

4 Vacuum energy

To calculate the vacuum energy of the deformed non-supersymmetric theory we will use the standard AdS/CFT technique [4]. The supergravity dual of the gauge theory Hamiltonian is a G_{00} component of the 10d metric. The vacuum energy, therefore, can be found by variation of the type IIB SUGRA action with respect to G_{00}. This variation vanishes on-shell, except a boundary term. Looking at the supergravity action, it is clear that the only such a boundary term will appear from the curvature part of the action. Since the vacuum energy does not depend on the world-volume coordinates we might consider the metric variation in the form $G_{00} \to q G_{00}$. Here we only quota the final result for the vacuum energy (see [15] for the derivation):

$$E \sim \lim_{\tau \to \infty} \left(e^{n(\tau)} \partial_\tau \ln h(\tau) \right), \qquad (24)$$

where $\bar{n} = -4\bar{q} + 4\bar{p} + 4\bar{Y}$. The divergent result we have found is expected to be canceled out when we compare the vacuum energies of our solution and of the KS background, which we take as a reference. Using that $h \to h_0 + \bar{h}$ and $n \to n_0 + \bar{n}$ we get:

$$\Delta E \sim \left[e^{n_0} \left(\partial_\tau \left(\frac{\bar{h}}{h_0} \right) + \bar{n} \partial_\tau \left(\ln h_0 \right) \right) \right]_{\tau \to \infty}, \qquad (25)$$

so that $\Delta E \sim \mu$. Here we used the asymptotic solutions for the fields at $\tau \to \infty$ from the previous section. In (25) the term $e^{n_0(\tau)}$ diverges at $\tau \to \infty$ as $e^{4\tau/3}$. This is suppressed by the $e^{-4\tau/3}$ term in the large τ expansion of the fields appearing in the parenthesis which multiply the $e^{n_0(\tau)}$ term. Furthermore, the term linear at τ cancels and we end up with a constant proportional to μ.

5 Dual gauge theory

As was announced in the introduction the deformation of the supergravity background corresponds in the gauge theory to an insertion of the soft supersymmetry breaking gaugino mass terms. The most general gaugino bilinear term has the form of $\mu_+ \mathcal{O}_+ + \mu_- \mathcal{O}_- + c.c$ where $\mathcal{O}_\pm \sim Tr[W^2_{(1)} \pm W^2_{(2)}]$ and $W_{(i)}$, $i = 1, 2$ relate to the $SU(N + M)$ and $SU(N)$ gauge groups respectively. Namely, the general deformation is characterized by two complex masses. Our non-supersymmetric deformation of the KS solution derived above is a special case that depends on only one real parameter μ. Since the supergravity identification of the operators $\mathcal{O}_\pm$ is known up to some constants of proportionality we can not determine the precise form of the soft symmetry breaking term.

In the non-deformed supersymmetric theory the $U(1)_R$ symmetry is broken [38], [39] first by instantons to $\mathbf{Z}_{2M}$ and then further spontaneously broken down to $\mathbf{Z}_2$ by a VEV of the gaugino bilinear. Let us discuss first the latter breaking. We have already seen that on the SUGRA side this fact is manifest from the UV behavior of the complex 3-form $G_3 = F_3 + \frac{i}{g_s} H_3$. The sub-leading term in the expansion of G_3 preserves only the $\mathbf{Z}_2$ part of the $U(1)_R$ symmetry and it vanishes at infinity like $e^{-\tau}$ matching the expectation from the scalar operator $\text{Tr}(\lambda\lambda)$ of dimension 3 with a non-zero VEV [25]. Plugging the non-supersymmetric solution into G_3 we find that the leading term breaking the $U(1)_R$ symmetry behaves like $\Delta G_3 = g(\tau) e^{-\tau/3}$, where $g(\tau)$ is some polynomial in τ. This is exactly what one would predict for an operator with $\Delta = 3$ and a non-trivial mass. The second combination of the gaugino bilinears is encoded in the 6d part of the metric. For the 6d metric in (9) to preserve the $U(1)_R$ one has to set $y = 0$. In the supersymmetric deformed conifold metric $y(\tau) = -2e^{-\tau} + \dots$ similarly to the behavior of the 3-form. In the non-supersymmetric solution $y(\tau)$ goes like $e^{-\tau/3}$ elucidating again that the gaugino combination gets a mass term. Notice also that the non-zero VEVs of the gaugino bilinears are modified by the SUSY breaking deformation. This is evident, for example, from the $Ve^{-\tau}$ term in the UV expansion of $\bar{y}(\tau)$ in (18). Clearly, for $V \neq 0$ we have a correction to the VEV in the supersymmetric theory which was encoded in the expansion of $y_0(\tau)$. Similar $e^{-\tau}$ term appears also in the expansion of $\xi_2(\tau)$ and therefore the VEV of the second combination of the gauginos gets modified too.

The spontaneous breaking of the $\mathbf{Z}_{2M}$ discrete group down to the $\mathbf{Z}_2$ subgroup by gaugino condensation results in an $\mathbf{M}$-fold degenerate vacua. This degeneracy is generally lifted by soft breaking mass terms in the action. For small enough masses one can treat the supersymmetry breaking as a perturbation yielding (for a single gauge group) the well-known result [40] that the difference in energy between a non-supersymmetric solution and its supersymmetric reference is given by $\Delta E \sim \mathrm{Re}(\mu C)$, where μ and C are the mass and the gaugino condensate respectively. For the gauge theory dual of the deformed KS solution the vacuum energy will in general be proportional to $\mathrm{Re}(a_+\mu_+ C_+ + a_-\mu_- C_-)$ where $C_\pm$ are the expectation values of $\mathcal{O}_\pm$ and $a_\pm$ are some proportionality constants. In the special deformation we are discussing in this paper this reduces to $\mu\mathrm{Re}(a_+ C_+ + a_- C_-)$. In the previous section we have derived a result using the SUGRA dual of the gauge theory which has this structure. For the softly broken MN background similar calculations were performed by [11]. In their case the explicit linear dependence on the condensate was demonstrated.

One of the properties of the supersymmetric gauge theory is the space-time independence of the correlation function of two gaugino bilinears. This appears from the supergravity dual description as follows [25]. Consider a perturbation of the complex 2-form:

$$C_2 \to C_2 + y\omega_2, \quad G_3 \to G_3 + y\omega_3 + \mathbf{d}y \wedge \omega_2, \tag{26}$$

where $\omega_{2,3}$ are given by (8) and $y(x,\tau)$ has non-vanishing boundary values. Plugging this forms into the relevant part of the type IIB action and integration over τ will *not* lead to a kinematic term $dy(x_1)dy(x_2)$ and therefore the corresponding correlation function will be space-time independent. This derivation is only schematic since there is a mixing between the 3-form modes and the modes coming from metric as we have seen in Section 3. Notice, however that this simplified calculation will yield the kinetic term for the deformed non-supersymmetric background, since the complex 3-form is not imaginary self dual in this case. Thus in the non-supersymmetric theory the correlation function will be time-space dependent as one would expect.

6 The plane wave limit

In this section we will construct a Penrose limit of the non-supersymmetric background. Following [22] we will expand the metric around a null geodesic that goes along an equator of the S^3 at $\tau = 0$. The parameter ε appearing in the 6d metric of the deformed conifold and the gauge group parameter $\mathbf{M}$ are both taken to infinity in the PL limit, while keeping finite the mass of the glue-ball: $M_{gb} \sim \frac{\varepsilon^{2/3}}{g_s\mathbf{M}\alpha'}$. The final result [15] is:

$$ds^2 = -4dx_-dx_+ + dx_i^2 + dz^2 + dud\bar{u} +$$
$$+dvd\bar{v} - m_0^2\Big[v\bar{v} + \Big(\Big(\frac{4a_1}{a_0} - \frac{4}{5}\Big) - 8\frac{3^{2/3}}{135}\mu\Big)z^2$$
$$+ \Big(\Big(\frac{4a_1}{a_0} - \frac{3}{5}\Big) + 4\frac{3^{2/3}}{135}\mu\Big)u\bar{u}\Big]dx_+^2, \tag{27}$$

where

$$m_0^2 = \frac{3^{1/3}\varepsilon^{4/3}}{2(g_s\mathbf{M}\alpha')^2 a_0}(1 + 2C_Y). \tag{28}$$

Recall that C_Y is a numerical constant proportional to μ. As expected for $\mu = 0$ we recover the result of the supersymmetric case [22]. We see that all the world-sheet masses $(m_v, m_z$ and $m_u)$ depend on the supersymmetry breaking parameter. Under the Penrose limit the 3-forms read:

$$(F_3)_{+v\bar{v}} = \Big(\frac{1}{3} + 4\gamma\Big)^{-1}(F_3)_{+u\bar{u}} = \frac{3im_0}{\sqrt{2}g_s}\sqrt{\frac{a_1}{a_0}}$$
$$(H_3)_{+u\bar{v}} = (H_3)_{+v\bar{u}} = \frac{im_0}{\sqrt{2}}\sqrt{\frac{a_1}{a_0}}(1 - 6\gamma).$$

7 The plane wave string theory and the Annulons

The string theory associated with the plane wave background described in the previous section is quite similar to that associated with the PL limit of the KS background. The bosonic sector includes three massless fields that correspond to the spatial directions on the world-volume of the D3 branes. Their masslessness is attributed to the translational invariance of the original metric and the fact that the null geodesic is at constant τ. The rest five coordinates are massive. The only difference between the bosonic spectrum of the deformed model and that of [22] is the shift of the masses of the $z, v, \bar{v}, u, \bar{u}$ fields. The sum of the $mass^2$, however, of the individual fields $\sum m^2 = 12m_0^2\frac{a_1}{a_0}$

still has the same form as the sum in the supersymmetric case apart from the modification of m_0 (28). The modification of m_0 is also responsible for the deviation of the deformed string tension with from the supersymmetric one since the string tension $T_s \sim g_s \mathbf{M} m_0^2$. The fermionic spectrum takes the form ($k = 1, \ldots, 4$ and $l = 1, 2$):

$$\omega_n^k \approx \sqrt{n^2 + \hat{m}_B^2 \left(1 + 18\gamma\right)},$$
$$\omega_n^l = \sqrt{n^2 + \frac{1}{4}\hat{m}_B^2} \pm \frac{1}{2}\hat{m}_B, \quad \text{where}$$
$$\hat{m}_B = \sqrt{2}p^+\alpha' m_0 \left(\frac{a_1}{a_0}\right)^{1/2} (1 - 6\gamma).$$

Comparing the bosonic and fermionic masses we observe that like in the undeformed KS model there is no linearly realized world-sheet supersymmetry and the hence there is a non-vanishing zero point energy. However, up to deviations linear in μ the sum of the square of the frequencies of the bosonic and fermionic modes match. Since this property follows in fact from the relation between R_{++} and $(G_3)_{+ij} (\bar{G}_3)_+^{ij}$ it should be a universal property of any plane wave background.

Surprisingly we find that the fermionic spectrum admits two fermionic zero modes $\omega_0^{l=1,2}$ exactly like in the supersymmetric case. The fermionic zero modes in the spectrum of the latter case were predicted [22] upon observing that the Hamiltonian still commutes with the four supercharges that correspond to the four dimensional $\mathcal{N} = 1$ supersymmetric gauge theory. This implies that four supersymmetries out of the sixteen supersymmetries of plane wave solution commute with the Hamiltonian giving rise to the four zero-frequency modes and a four dimensional Hilbert sub-space of (two bosonic and two fermionic) degenerate states. One might have expected that in the PL of the deformed theory the fermionic zero modes will be lifted by an amount proportional to the supersymmetry breaking parameter. Our results, however, contradict this expectation. In the dual field theory this implies that even though the full theory is non-supersymmetric, the sector of states with large J charge admits supersymmetry. As will be discussed below these states are characterized by their large mass which is proportional to J. Presumably, in this limit of states of large mass

the impact of the small gaugino mass deformations is washed away. For instance one can estimate that the ratio of the boson fermion mass difference to the mass of the annulon scales like $\frac{\mu}{J}$ and since μ has to be small and $J \to \infty$ this ratio is negligible.

Note that the fermionic zero modes are in accordance with the the criteria presented in [23]. However, the metric and the 3-form given in [23] do not coincide with our results, because of the factor of C_Y in the expression for m_0^2.

Since apart from the modifications of the fermionic and bosonic frequencies the string Hamiltonian we find has the same structure as the one found for the KS case, the analysis of the corresponding gauge theory states also follows that of [22]. We will not repeat here this analysis, but rather just summarize its outcome:

- The ground state of the string corresponds to the *Annulon*. This hadron which carries a large J charge is also very massive since its mass is given by

$$M_{annulon} = m_0 J \qquad (29)$$

Obviously, the only difference between the annulon of the deformed theory in comparison with the supersymmetric one is the modification of m_0.

- The annulon can be described as a ring composed of J constituents each having a mass (in the mean field of all the others) of m_0.

- The annulon which is a space-time scalar has a fermionic superpartner of the same mass. The same holds for the rest of the bosonic states.

- The string Hamiltonian has a term $\frac{P_i^2}{2m_0 J}$ that describes a non-relativistic motion of the annulons.

- The annulons admit stringy ripples. The spacing between these excitations are proportional to $\frac{T_s}{M_{annulon}}$.

- The string Hamiltonian describes also excitations that correspond to the addition of small number of different constituents on top of the J basic ones.

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